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Dynamics of a string coupled to gravitational waves

— Gravitational wave scattering by a Nambu-Goto straight string —

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Abstract

We study the perturbative dynamics of an infinite gravitating Nambu-Goto string within the general-relativistic perturbation framework. We develop the gauge invariant metric perturbation on a spacetime containing a self-gravitating straight string with a finite thickness and solve the linearized Einstein equation. In the thin string case, we show that the string does not emit gravitational waves by its free oscillation in the first order with respect to its oscillation amplitude, nevertheless the string actually bends when the incidental gravitational waves go through it.

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There are significant interests in topological defects formed during phase transition in the early universe [1,2]. In particular, it has been thought that these defects radiate gravitational wave by their rapid oscillation [3]. Thus, it is crucially important to study the precise dynamics of the defects and their gravitational effects.

In the simplest case, the defects are idealized by the infinitesimally thin Nambu-Goto membranes, i.e., their dynamics is governed by the minimization of their world hyper-volume. If the self-gravity of membranes is ignored (test membrane case), the Nambu-Goto action admits oscillatory solutions. It is considered that the membranes gradually lose their kinetic energy by the gravitational wave emission [3]. However, by taking into account the self-gravity of the Nambu-Goto wall, it is shown that a self-gravitating wall coupled to gravitational wave behaves in a quite different manner [4]. The dynamical degree of freedom concerning the perturbative oscillations around a spherical one is given by that of gravitational waves and self-gravitating spherical walls do not oscillate spontaneously unlike test walls.

How about Nambu-Goto strings? It is also considered that the cosmic strings oscillate rapidly and gradually lose their kinetic energy by the gravitational wave emission [3]. The energy momentum tensor of an oscillating infinite test string and the gravitational wave emission are studied by several authors [5,6]. However, it is not clear how a Nambu-Goto string behaves when one takes into account of its self-gravity.

In this paper, we consider the perturbative oscillation of an infinite self-gravitating string using a exactly solvable model within the general-relativistic perturbation framework and show that a self-gravitating infinite string behaves in the same manner as the above self-gravitating wall in the first order with respect to the oscillation amplitude of the string.

It is known that the mathematical description of a thin string is more delicate than a domain wall because the support of a string is a surface of co-dimension two. There is no simple prescription of an arbitrary line source where a metric becomes singular [7]. In this paper, we consider, first, a straight string with a finite thickness so that the singularity is regularized [1,8]. Then, the metric junction formalism is applicable on the surface of the

thick string. Next, we consider gravitational wave emission by the thick string motion which is excited by incident gravitational wave, i.e., the scattering problem of gravitational wave by a thick string. We analyze this problem by the gauge invariant linear perturbation theory and show the perturbative velocity of the string is given by the variable of gravitational waves. In the thin string case, we show that there is neither resonance nor phase shift in the gravitational wave scattered by a Nambu-Goto string, nevertheless the string is bent by gravitational waves. This shows that self-gravitating infinite strings do not oscillate spontaneously unlike test string at least in the first order with respect to its oscillation amplitude.

As the background for the perturbation, we consider a spacetime $(\mathcal{M}, g_{\mu\nu})$ containing a straight thick string. The surface $\mathcal{S}$ of the thick string divides $\mathcal{M}$ into two regions: $\mathcal{M}_{ex}$ and $\mathcal{M}_{in}$. Note that $\mathcal{M}_{in}$ describes the 'thick' worldsheet of the string. We assume that the spacetime $\mathcal{M}$ is static and cylindrically symmetric. Then we divide $\mathcal{M}$ into two submanifolds so that $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$ and write the background metric on $\mathcal{M}$ in the form

$$ds^2 = \gamma_{ab} dy^a dy^b + \eta_{pq} dz^p dz^q, \tag{1}$$

where γ_{ab}, the metric on $\mathcal{M}_1$, and η_{pq}, that on $\mathcal{M}_2$, are given by

$$\gamma_{ab} dy^a dy^b = d\rho^2 + r(\rho)^2 d\phi^2, \quad \eta_{pq} dz^p dz^q = -dt^2 + dz^2, \quad 0 \leq \phi \leq 2\pi. \tag{2}$$

We shall use the indices $a, ..., d$ for tensors on $\mathcal{M}_1$ and $p, ..., s$ for those on $\mathcal{M}_2$. The string thickness is given by the circumference radius r_* of $\mathcal{S}$.

Since a Nambu-Goto string is characterized by a constant string tension σ_0, we consider the following energy-momentum tensor,

$$T_{\mu\nu} = -\sigma \eta_{\mu\nu}, \tag{3}$$

where $\sigma = \sigma_0$ for $r < r_*$ and $\sigma = 0$ for $r \geq r_*$ [1]. Here $\eta_{\mu\nu}$ is the four dimensional extension of η_{pq}.

The Einstein equations for the metric (1) and (2) are reduced to the single equation

$$\mathcal{R} = -2\frac{\partial_\rho^2 r}{r} = 16\pi G\sigma, \tag{4}$$

where $\mathcal{R}$ is the Ricci curvature on $\mathcal{M}_1$. The solution on $\mathcal{M}_{in} \cap \mathcal{M}_1$ is given by

$$\gamma_{ab}dy^a dy^b = \frac{dr^2}{1-\hat{\alpha}^2 r^2} + r^2 d\phi^2, \quad \hat{\alpha}^2 = \frac{\mathcal{R}}{2}, \tag{5}$$

and that on $\mathcal{M}_{ex} \cap \mathcal{M}_1$ is

$$\gamma_{ab}dy^a dy^b = \frac{dr^2}{(1-\alpha)^2} + r^2 d\phi^2, \tag{6}$$

where α is a deficit angle on $\mathcal{M}_{ex}$. These two solutions are joined along the surface $\mathcal{S}$ by Israel's junction condition [9]: $[K^\mu{}_\nu] := K^\mu{}_{\nu+} - K^\mu{}_{\nu-} = 0$, where $K^\mu{}_{\nu\pm}$ are the extrinsic curvature of $\mathcal{S}$ facing to $\mathcal{M}_{ex}$ and $\mathcal{M}_{in}$, respectively. For the solutions (5) and (6), this junction condition is reduced to $\alpha = 1-\sqrt{1-\hat{\alpha}^2 r_*^2}$. The global geometry of $\mathcal{M}_1$ is illustrated in Fig.1.

We consider the metric perturbations on the background geometry given by (1), (5) and (6). Let $h_{\mu\nu}$ be a perturbative metric and $t^\mu{}_\nu$ be a perturbed energy-momentum tensor, which can be expanded by the harmonics on $\mathcal{M}_2$ as follows:

$$h_{ab} = \int f_{ab} S, \quad h_{ap} = \int \left\{ f_{a(o1)} V_{(o1)p} + f_{a(e1)} V_{(e1)p} \right\}, \tag{7}$$

$$h_{pq} = \int \left\{ f_{(o2)} T_{(o2)pq} + f_{(e0)} T_{(e0)pq} + f_{(e2)} T_{(e2)pq} \right\}, \tag{8}$$

$$t^a{}_b = \int s^a{}_b S, \quad t^a{}_p = \int \left\{ s^a_{(o1)} V_{(o1)p} + s^a_{(e1)} V_{(e1)p} \right\}, \tag{9}$$

$$t^p{}_q = \int \left\{ s_{(o2)} T_{(o2)}{}^p{}_q + s_{(e0)} T_{(e0)}{}^p{}_q + s_{(e2)} T_{(e2)}{}^p{}_q \right\}. \tag{10}$$

Here $\int := \int d\omega dk_z$ and

$$S := e^{-i\omega t + ik_z z}, \qquad V^p_{(o1)} := \epsilon^{pq}\hat{D}_q S, \qquad V^p_{(e1)} := \eta^{pq}\hat{D}_q S,$$
$$T_{(e0)pq} := \frac{1}{2}\eta_{pq} S, \quad T_{(e2)pq} := \left(\hat{D}_p\hat{D}_q - \tfrac{1}{2}\eta_{pq}\hat{D}^r\hat{D}_r\right) S, \quad T_{(o2)pq} := -\epsilon_{r(p}\hat{D}_{q)}\hat{D}^r S, \tag{11}$$

are independent tensor harmonics on $\mathcal{M}_2$, ϵ_{rs} is a two-dimensional antisymmetric tensor on $\mathcal{M}_2$, and $\hat{D}_p$ denotes the covariant derivative associated with η_{pq}. The symbols (o) and (e) refer to odd and even parity modes with respect to the inversion of (t, z), respectively. The

expansion coefficients are tensors on $\mathcal{M}_1$. The perturbative energy momentum tensor $t^\mu{}_\nu$ has its support only on $\mathcal{M}_{in}$.

We define $\kappa^2 := \omega^2 - k_z^2$, which is the eigen value of a differential operator $\eta^{pq}\hat{D}_p\hat{D}_q$. We note that the mode with $\kappa = 0$, which propagates along the string, is not included in the expansion (7)-(10). The $\kappa = 0$ mode will be discussed in Ref. [10].

Here we consider the gauge-transformation of $h_{\mu\nu}$ and $t^\mu{}_\nu$ associated with $x^\mu \to x^\mu + \xi^\mu$, where ξ^μ is expanded as

$$\xi_a := \int \zeta_a S, \quad \xi_p := \int \left\{ \zeta_{(o1)} V_{(o1)p} + \zeta_{(e1)} V_{(e1)p} \right\}. \tag{12}$$

Inspecting the gauge transformed variables $h_{\mu\nu} - \pounds_\xi g_{\mu\nu}$ and $t^\mu{}_\nu - \pounds_\xi T^\mu{}_\nu$, we find simple gauge-invariant combinations of the expansion coefficients: for odd modes,

$$F_a := f_{a(o1)} - \frac{1}{2} D_a f_{(o2)}, \tag{13}$$

and for even modes,

$$F_{ab} := f_{ab} - D_a X_b - D_b X_a, \quad F := f_{(e0)} - \kappa^2 f_{(e2)}, \tag{14}$$

where D_a is a covariant derivative associated with γ_{ab} and the variable $X^a := f^a_{(e1)} - \frac{1}{2} D^a f_{(e2)}$ is transformed to $X^a - \zeta^a$ by the gauge transformation.

We also introduce the gauge invariant variables for the perturbations of $T^\mu{}_\nu$ by

$$\Sigma := 16\pi G(s_{(e0)} + 2X^a D_a \sigma), \quad V^a := 16\pi G(s^a_{(e1)} - \sigma X^a). \tag{15}$$

Note that all the expansion coefficients except for $s_{(e0)}$ and $s^a_{(e1)}$ are gauge invariant by themselves. In this article, we consider the perturbative motion of a Nambu-Goto string in the first order with respect to its oscillation amplitude. Within this order, Σ is the energy density perturbation which is equal to the tangential tension of a string and V^a corresponds to the momentum perturbation. The other coefficients, within the same order, $s^a{}_b$, $s^a_{(o1)}$, $s_{(o2)}$ and $s_{(e2)}$ are regarded as the tension normal to the string worldsheet, spin of the string, Lorentz boost along the string and the energy density perturbation which is not equal to the tangential tension, respectively.

As derived in [5,6], Σ and V^a are induced by the motion of an infinite Nambu-Goto string within the first order of oscillation amplitude, while the others are induced in the higher order. Their results show that the energy momentum perturbations $s^a{}_b$, $s^a_{(o1)}$, $s_{(o2)}$ and $s_{(e2)}$ are irrelevant to the perturbation of an infinite Nambu-Goto string within the first order. This corresponds to the fact that a Nambu-Goto string is only charactorized by its energy density equal to its tangential tension and does not have any other properties such as the tension normal to its worldsheet, spin or boost along itself. Hence, in this paper, we concentrate only on Σ and V^a and drop the other coefficients in the perturbative energy momentum tensor (9) and (10), since we consider the dynamics of an infinite Nambu-Goto string within the first order of its oscillation amplitude.

In terms of the gauge invariant variables, we write the perturbed Einstein equations: for odd modes,

$$D^a F_a = 0, \quad (\Delta + \kappa^2) F_a - D^c D_a F_c = 0, \tag{16}$$

and for even modes,

$$(\Delta + \kappa^2) F_{ab} = \mathcal{R} F_{ab} + 2 D_{(a} V_{b)} - \gamma_{ab} D_c V^c, \quad (\Delta + \kappa^2) F = 0, \tag{17}$$

$$D^c F_{ac} - \frac{1}{2} D_a F = V_a, \quad F^c{}_c = 0, \tag{18}$$

where $\Delta := D^a D_a$. The perturbative divergence of the energy momentum tensor are reduced to

$$\kappa^2 V^a + \frac{1}{2} \mathcal{R} D^a F = 0, \quad D_a V^a + \frac{1}{2} \Sigma = 0. \tag{19}$$

The first equation corresponds to the Euler equation which coincides with the equation of the perturbative string motion derived from the Nambu-Goto action [10] and the second equation corresponds to the continuity equation for the energy density.

We find that (16)-(19) for even and odd modes on $\mathcal{M}_{in}$ are reduced to the wave equations for two scalar variables $\Phi_{(o)}$ and $\Phi_{(e)}$

$$(\Delta + \kappa^2) \Phi_{(o),(e)} = 0, \tag{20}$$

respectively, and all gauge invariant variables are given by $\Phi_{(o)}$ and $\Phi_{(e)}$ without loss of generality as follows [10]:

$$F_a = \epsilon_{ab} D^b \Phi_{(o)}, \tag{21}$$

$$F_{ab} = \left(D_a D_b - \frac{1}{2}\gamma_{ab}\Delta \right) \Phi_{(e)}, \quad F = \Delta\Phi_{(e)}, \tag{22}$$

$$V_a = \frac{1}{2}\mathcal{R} D_a \Phi_{(e)}, \quad \Sigma = -\mathcal{R}\Delta\Phi_{(e)}, \tag{23}$$

where ϵ^{ab} is the two-dimensional antisymmetric tensor on $\mathcal{M}_1$. On $\mathcal{M}_{ex}$, we also find that (16)-(18) are reduced to the same form as (20)-(23) with $\mathcal{R} = 0$. The exterior solution $\Phi^{(ex)}_{(o),(e)}$ and the interior solution $\Phi^{(in)}_{(o),(e)}$ to (20) are

$$\Phi^{(ex)}_{(o),(e)} = \sum_{m=0}^{\infty} e^{im\phi} \left\{ A H^{(1)}_{\mu}(\beta r) + B H^{(2)}_{\mu}(\beta r) \right\}, \tag{24}$$

$$\Phi^{(in)}_{(o),(e)} = \sum_{m=0}^{\infty} e^{im\phi} \left\{ C P^m_{\nu}(x) + D Q^m_{\nu}(x) \right\}, \tag{25}$$

where $H^{(1)}_{\mu}(\beta r)$ and $H^{(2)}_{\mu}(\beta r)$ are the Hankel function of the first and the second class, $P^m_{\nu}(x)$ and $Q^m_{\nu}(x)$ are the associated Legendre function of the first and second class, and $\mu := m/(1-\alpha)$, $\beta := \kappa/(1-\alpha)$, $\nu(\nu+1) := \kappa^2/\hat{\alpha}^2$ and $x := \sqrt{1-\hat{\alpha}^2 r^2}$. The coefficients A and B correspond to the amplitude of the outgoing and the incident wave, respectively. The regularity condition at the axis $r = 0$ on $\Phi^{(in)}_{(o),(e)}$ leads $D = 0$.

Now, we construct the global solutions to the perturbed Einstein equations in $\mathcal{M}$ by matching the exterior and the interior solutions (24) and (25) along the thick string surface $\mathcal{S}$ $(r = r_*)$. The perturbed solutions should satisfy the perturbed junction conditions $[\delta q_{\mu\nu}] = 0$ and $[\delta K^{\mu}{}_{\nu}] = 0$, where $\delta q_{\mu\nu\pm}$ is the perturbed intrinsic metric and $\delta K^{\mu}{}_{\nu\pm}$ is the perturbed extrinsic curvature of $\mathcal{S}$. These perturbed quantities are described by the metric perturbations, which are represented by $\Phi_{(e),(o)}$ through (21) and (22). After some calculations, the perturbed junction conditions tell us

$$[\Phi_{(o,e)}] = 0, \quad [D_{\perp}\Phi_{(o,e)}] = 0, \tag{26}$$

where $D_{\perp} = n^a D_a$ and $n^a = (\partial/\partial\rho)^a$.

Substituting (24) and (25) into (26), we have

$$A = UB, \tag{27}$$

$$U = -\frac{\kappa r_* H^{(2)}_{\mu+1}(\beta r_*) P^m_\nu(x_*) - \left(\sqrt{1-x_*^2} P^{m+1}_\nu(x_*) + \alpha m P^m_\nu(x_*)\right) H^{(2)}_\mu(\beta r_*)}{\kappa r_* H^{(1)}_{\mu+1}(\beta r_*) P^m_\nu(x_*) - \left(\sqrt{1-x_*^2} P^{m+1}_\nu(x_*) + \alpha m P^m_\nu(x_*)\right) H^{(1)}_\mu(\beta r_*)}, \tag{28}$$

where $x_* := \sqrt{1-\hat{\alpha}^2 r_*^2} = 1-\alpha$. The absolute value of U with $m = 1$ and $\alpha = 0.3$ is illustrated in Fig.2.

The deformation of $\mathcal{S}$ is represented by V^a on $\mathcal{S}$. By the appropriate choice of the function ζ^a, we fix the gauge freedom $X^a \to X^a - \zeta^a$ in the neighborhood of $\mathcal{S}$ so that

$$X_a|_{\mathcal{S}} := X_{a\pm} = -\frac{1}{\mathcal{R}} V_a\Big|_{\mathcal{S}-} = -\frac{1}{2} D_a \Phi_{(e)}\Big|_{\mathcal{S}}. \tag{29}$$

Since $i\int V_a \omega S$ is a precise momentum perturbation, $\int X_a|_{\mathcal{S}}\, S$ does represent the deformation of $\mathcal{S}$.

Now, we consider the thin string case. Physically, a "thin string" means a string whose thickness r_* is sufficiently smaller than the wavelength of gravitational wave. In this paper, we consider the situation $\epsilon := \beta r_* \ll 1$ with the finite outside deficit angle α and take the leading order of ϵ for the thin string case.

Further, we note that only $m = 1$ mode in (24) and (25) shows the motion of a Nambu-Goto thin string. $m = 0$ and $m > 1$ modes are irrelevant to a thin string [10]. Hence, in the thin string case, the displacement X^a_S of a Nambu-Goto string by the gravitational wave scattering is given by

$$X^a_S := \int X^a|_{\mathcal{S}}\,(m = 1, \epsilon \ll 1) S = \int \frac{\kappa B_{m=1} S}{2(1-\alpha)\Gamma(\frac{2-\alpha}{1-\alpha})} \left(\frac{\epsilon}{2}\right)^{\frac{\alpha}{1-\alpha}} e^{i\phi}\,(n^a + i\tau^a)\,, \tag{30}$$

where $\tau^a = (1/r)(\partial/\partial\phi)^a$ and $n^a = (1-\alpha)(\partial/\partial r)^a$. (30) shows that the string is deformed while the incident wave exists on the string. In the same order calculation where X^a_S is given by (30), we obtain the trivial scattering data $U \sim 1$ from (28).

The order of magnitude of $|X^a_S| := \sqrt{\gamma_{ab} X^a_S X^b_S}$ is estimated as follows:

$$|X^a_S|/r_* \sim \kappa^2 |B_{m=1}| \epsilon^{\alpha/(1-\alpha)}/(\kappa r_*) \sim \kappa^2 |B_{m=1}| \epsilon^{\alpha/(1-\alpha)-1}. \tag{31}$$

The amplitude of gravitational wave (F or F_{ab}) is proportional to $|\kappa^2 B| \ll 1$. If $\epsilon^{1-\alpha/(1-\alpha)} < \kappa^2|B| \ll 1$, then $|X^a_S| > r_*$. Therefore, the magnitude of the displacement may become larger than the string thickness within the linear perturbation framework.

Thus, we have obtain the result that there is neither resonance nor phase shift in the scattering problem, nevertheless the string is deformed by the gravitational waves. In particular, the fact that there is no resonance means an infinite thin string does not emit gravitational wave spontaneously by its oscillatory motion. It should be noted that the trivial scattering does not dictate the absence of gravitational lensing effect by the deficit angle α on $\mathcal{M}_{ex}$. We will explicitly see the lensing effect by the scattering of wave packet which is suitably constructed by (24) because the mode functions already include the effect of α.

Using the linearized Einstein equation, we have found that the perturbative string displacement X^a_S is represented by the gravitational wave $\Phi_{(e)}$. In this sense, the dynamical degree of freedom of the string displacement is given by that of the gravitational waves on the string surface. Further, (29) and the trivial scattering data show that the string is bent while the incident wave is passing through the string worldsheet. These behaviors are same as that for a self-gravitating spherical Nambu-Goto wall in the first order with respect to its oscillation amplitude [4]. This is our main conclusion.

To obtain the above results, we have first considered the scattering by a thick string. We regard that a "thin string" is not a string with the thickness $r_* \to 0$ but that whose thickness r_* is sufficiently smaller than the wavelength of gravitational wave. For a fixed amplitude of the incident wave, the magnitude of the string displacement X^a_S depends on r_*. If we take the limit $r_* \to 0$ for fixed the wavelength of gravitational wave, these is no response of string motion for finite incident gravitational wave. This result is consistent with that obtained by Unruh et.al. [11]. Both their and our results show that the straight string cannot bend in the limit $r_* \to 0$. This mathematical limit will be irrelevant for strings formed during phase transition in the early universe, because they have finite thickness. Further, the scattering data (28) has the resonance poles at $\beta r_* \sim 1$ or larger as seen in Fig.2. This suggest that a

cosmic string oscillates spontaneously and emits the gravitational waves with the frequencies of the order of the string thickness. In this situation, the dynamics of strings is no longer approximated by that of thin Nambu-Goto strings.

One might think that our result depends sensitively on the distribution of σ in (3) and our model would be too artificial because of the step function distribution of σ. Further, the above analysis does not include $\kappa = 0$ mode which includes "cosmic string traveling waves" discussed in Ref. [6,12], and one might think that the dynamical degree of freedom of the string spontaneous oscillation is in this $\kappa = 0$ mode. However, we obtain the same conclusion in the thin string case even when the background σ is different from the step function and our conclusion is unchanged in the thin string case even if we include the $\kappa = 0$ mode into our consideration. These two points will be discussed in a separated paper [10].

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FIGURES

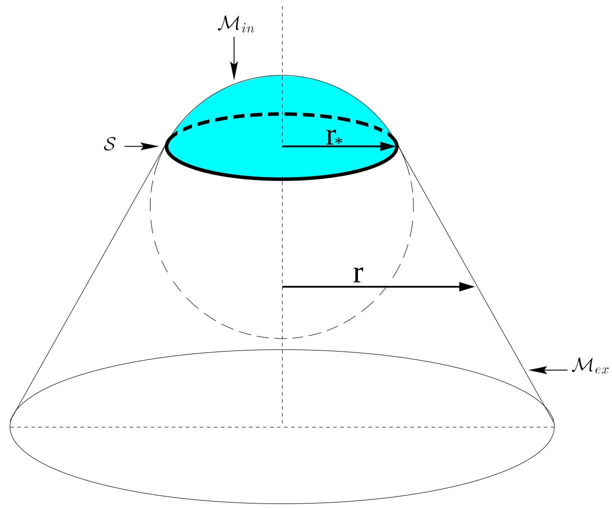

FIG. 1. The global geometry of $\mathcal{M}_1$.

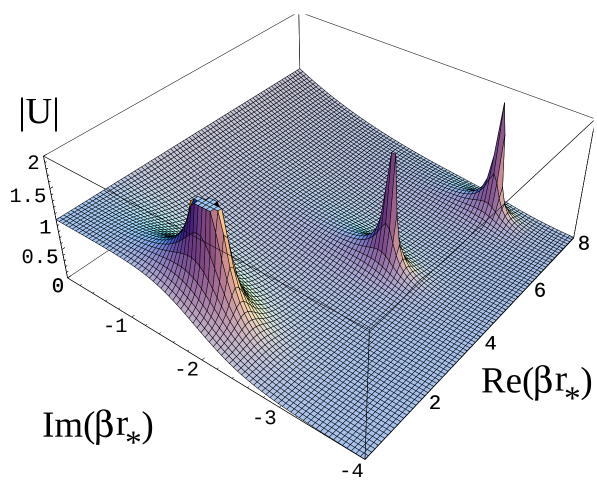

FIG. 2. The absolute value of the scattering data U with $\alpha = 0.3$ and $m = 1$ in the complex βr_* plane is illustrated. There are some resonance poles associated with the string thickness.